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Forbidden subgraph decomposition

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Abstract

We define a new form of graph decomposition, based on forbidding a fixed bipartite graph from occurring as an induced subgraph of edges which cross a partition of the vertices. We show that the generalized join decomposition proposed by Hsu (J. Combin. Theory Ser. B 43 (1987) 70) is NP-complete, while some other decompositions which can be described in this way can be found in polynomial time. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

This paper introduces a new notion of decomposing a graph. For any bipartite graph F, we define an F-free decomposition, which corresponds to partitioning the vertices into two subsets such that F does not occur as an induced subgraph of the edges which go from one subset to the other.

We study the complexity of finding a decomposition for small F, and a number of other questions which arise while studying these decompositions. Among other results, we show that it is NP-complete to determine whether a graph has a generalized join decomposition, thus resolving an open problem posed by Hsu [8].

The remainder of this section is devoted to definitions which allow new notions regarding these decompositions to be specified precisely. Later sections will be devoted to F-free decompositions with respect to certain specific graphs F.

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For an arbitrary graph G, a partition P of its vertices into sets V_1, V_2 is denoted as (V_1, V_2). Throughout the paper, a partition will be *ordered*, that is, (V_1, V_2) and (V_2, V_1) will be distinct partitions.

Let G_P be the bipartite graph with bipartition V_1, V_2, and an edge from $v_1 \in V_1$ to $v_2 \in V_2$ if and only if v_1 is adjacent to v_2 in G. Let F be a bipartite graph with (ordered) bipartition (X, Y).

We say that F is *tidily induced* in G_P if there exists an isomorphism f from F to some induced subgraph H of G_P, such that $f(x) \in V_1$ for all $x \in X$ and $f(y) \in V_2$ for all $y \in Y$. When no confusion is possible, we will skip the word "tidily".

The partition P is called an *F-free decomposition* of G if it meets the following criteria:

(1) $|V_1| \geqslant |X|$, $|V_2| \geqslant |Y|$;
(2) F is not tidily induced in G_P.

Moreover, P is called a *symmetric-F-free decomposition* if both (V_1, V_2) and (V_2, V_1) are F-free decompositions.

Of particular interest (because of their strong properties) are the graphs for which the decomposability is a hereditary property. A graph G is *totally decomposable* with respect to the F-free decomposition if every induced subgraph of G with at least $|X| + |Y|$ vertices has an F-free decomposition.

It seems natural to focus our first efforts on studying F-decompositions for small fixed graphs F. As we will show later, certain decompositions defined in this way correspond naturally to classical graph decomposition techniques from the literature.

Forbidden subgraphs defined by two vertices are too limited to consider; forbidding a single edge allows only division into disconnected sets, while forbidding the two vertex graph with no edges allows only division into disconnected sets of $\bar{G}$. Totally decomposable graphs with respect to these two decompositions are simply edgeless graphs, and cliques.

We consider now the forbidden subgraphs with three vertices.

2. Three vertex forbidden subgraphs

Assuming without loss of generality that F (with the bipartition (X, Y) as before) has $|X| = 2$, $|Y| = 1$, there are only two forbidden subgraphs to consider separately; K_2 plus an isolated vertex in X, and $K_{1,2}$; the edgeless graph on three vertices is simply the bipartite complement of $K_{1,2}$, which corresponds to decomposing the complement graph.

Forbidding K_2 plus an isolated vertex is equivalent to the well-known notion of a module in the substitution decomposition [9]. We briefly present it in the next subsection. Then we show that $K_{1,2}$-free decompositions can be found in polynomial time. Chvatal [1] showed that it is NP-complete to determine whether G has a symmetric-$K_{1,2}$-free decomposition.

2.1. Edge plus isolated vertex

It is interesting that the decomposition defined by forbidding $F = K_2 + K_1$ already gives us the notion of a module, which is the fundamental subgraph for the well-known substitution decomposition.

A *module* in G is a set M of vertices which is 'indistinguishable' from outside of M; i.e. for every vertex v of $V - M$, v is either adjacent to every vertex of M, or v is adjacent to no vertex in M. A module M is called *trivial* if $M = V$, or $|M| = 1$.

Any nontrivial module can be chosen to be V_1 to create an F-free decomposition of G, and for every F-free decomposition, V_1 must form a nontrivial module. Thus, it is possible to determine whether G has an F-free decomposition in linear time, though this is by no means trivial [11,3,5]. Using well-known facts about the substitution decomposition [15], a graph is completely decomposable with respect to F-free decomposition if and only if it contains no P_4; in other words, if G is a cograph.

A graph is symmetrically decomposable with respect to F if and only if it can be partitioned into two modules of size at least two. This is possible if and only if either G or $\bar{G}$ is disconnected and the decomposition does not consist of a component of size $n+1$ and a component of size $n-1$. This condition can easily be tested in linear time. Completely decomposable graphs for the symmetric decomposition are simply cliques and independent sets.

2.2. $K_{1,2}$-free decomposition and star-free decomposition

We give a polynomial time algorithm for $K_{1,2}$-free decomposition, and then observe that a similar approach is valid for $K_{1,i}$-free decompositions, for any value of i.

Note that the meaning of the $K_{1,2}$-free decomposition is that vertices are divided into sets V_1 and V_2 so that every vertex in V_1 has at most one neighbor in V_2.

Our first remark is that this decomposition can be found by a forcing algorithm in $O(n^2 m)$ time. Indeed, for every pair of vertices x, y, we can find a decomposition with x and y in V_2 in $O(n+m)$ time, if such a decomposition exists. We mark all vertices not currently in V_2 with their number of neighbors in V_2; whenever a vertex gets a count of 2, we must bring it into V_2. If we stop before bringing every vertex into V_2, we have found a valid decomposition. This procedure looks at every edge once, and is thus $O(m)$ for a particular choice of x and y, giving an $O(n^2 m)$ algorithm for determining whether G is decomposable.

It is also easy to see that G is totally decomposable if and only if G is triangle-free. Since a triangle cannot be decomposed, every totally decomposable graph must be triangle-free. If G is triangle-free, we can make any adjacent vertices x, y our set V_2; since G is triangle-free, no vertex is adjacent to both x and y, so this decomposition is valid.

Clearly, the same type of algorithm will work to find $K_{1,i}$-free decompositions for any value of i in $O(n^i \mathrm{m})$ time; we start with each set of size i in V_2, and see whether all vertices are forced in V_2 in $O(m)$ time.

It is somewhat less obvious that the totally decomposable graphs for $K_{1,i}$-free decomposition are exactly those graphs without cliques of size $i+1$. One direction is clear, since a clique of size $i+1$ cannot be decomposed. To see that every graph without a clique of size $i+1$ can be decomposed, we consider two possibilities.

Let G_v be the graph obtained from G by removing the vertex v and all its incident edges. If G_v has a $K_{1,i-1}$-free decomposition, then v can be added to V_2 to obtain a $K_{1,i}$-free decomposition. Otherwise, we may assume that G_v has a clique of size i (using induction on the fact that totally $K_{1,i}$-free decomposable graphs are exactly those without cliques of size $i+1$). We place this i-clique in V_2, and all other vertices in V_1; since there is no clique of size $i+1$, this must give us a $K_{1,i}$-free decomposition.

2.3. Symmetric $K_{1,2}$-free decomposition

This very natural form of decomposition, requiring that the edges connecting the two sets are a matching, cannot be found in polynomial time unless $\mathrm{P}=\mathrm{NP}$. This will be the key to showing that decomposing with respect to other forbidden subgraphs is NP-complete.

Theorem 1 (Chvátal [1]). *It is NP-complete to determine whether a graph G has a symmetric $K_{1,2}$-free decomposition.*

It is easy to see that a graph on at least four vertices which is totally decomposable with respect to symmetric $K_{1,2}$-free decomposition can contain no K_3 and no $K_{1,3}$. Conversely, every (K_3, $K_{1,3}$)-free graph with four vertices or more is a collection of paths and cycles. Two adjacent vertices of degree two can be put on one side of the partition, while all the other vertices are on the other side.

3. Four vertex forbidden subgraphs

When F has four vertices, there are (up to symmetries and complementation) six cases to consider. One of them, $K_{1,3}$, was already discussed in Section 2. We will show that recognizing decomposable graphs with respect to $F=K_{2,2}$ and $2K_2$ is NP-complete; the first result is easy, while the second corresponds to the generalized join decomposition and is the main result of this paper. We leave the other three cases open. Two of these cases correspond to various bipartitions of P_3+K_1. The last open case is $F=P_4$, which we discuss briefly below.

Before discussing P_4-free decomposition, let us recall the split-decomposition [2]. This is a partition of the graph G into two sets V_1, V_2 such that there exist sets $W_1 \subseteq V_1$, $W_2 \subseteq V_2$ with the property that every vertex in W_1 is adjacent to every vertex in W_2 and no other edge exists between a vertex in V_1 and a vertex in V_2.

It is easy to see that a partition is a split-decomposition if and only if it is both a P_4-free decomposition and a $2K_2$-free decomposition. Finding a split decomposition can be done in linear time [4], and the totally decomposable graphs are in this case the distance-hereditary graphs [7].

When we drop the condition on the $2K_2$-freeness of the decomposition, we obtain the P_4-free decomposition, which may be expressed in terms of sets by asking that sets $W_1^i \subseteq V_1$, $W_2^i \subseteq V_2$ $(i=1,2,\ldots,k)$ exist such that every vertex in W_1^i is adjacent to every vertex in W_2^i and no other edge exists between a vertex in V_1 and a vertex in V_2. This equivalent formulation is easy to obtain by noticing that any two vertices in V_1 (respectively, in V_2) must have in V_2 (respectively, in V_1) either the same neighborhood, or disjoint neighborhoods.

In the opinion of the authors, determining whether a graph is P_4-free decomposable in polynomial time is the most natural single decomposition left open in this paper.

3.1. Forbidding $K_{2,2}$

It is easy to show that this problem is NP-complete, using the fact that determining whether G has a symmetric $K_{1,2}$-free decomposition is NP-complete.

Theorem 2. *It is NP-complete to determine whether a graph G has a $K_{2,2}$-free decomposition.*

Proof. The problem is clearly in NP. To prove NP-completeness, we reduce the symmetric $K_{1,2}$-free decomposition to the $K_{2,2}$-free decomposition. To do this, just add to an instance G of the first problem two nonadjacent vertices x, y which are adjacent to every vertex of G. Call this new graph G'.

Now, if G has a symmetric $K_{1,2}$-free decomposition (V_1, V_2), then the decomposition $(V_1 \cup \{x\}, V_2 \cup \{y\})$ is a $K_{2,2}$-free decomposition of G'; otherwise, the $K_{2,2}$ induced by $a, b \in V_1 \cup \{x\}$ and $c, d \in V_2 \cup \{y\}$ would contain at most one vertex among x, y (since $xy \notin E$) and the three other vertices would induce a $K_{1,2}$ or $K_{2,1}$ in G.

Conversely, if G' has a $K_{2,2}$-free decomposition (W_1, W_2), then x, y are on different sides of the partition (else they would form a $K_{2,2}$ with two arbitrary vertices on the other side of the partition). Say $x \in W_1, y \in W_2$. Then $(W_1 - \{x\}, W_2 - \{y\})$ is a symmetric $K_{1,2}$-free partition of G. □

We leave the question of characterizing totally decomposable graphs with respect to $K_{2,2}$-free decomposition as an open problem. Clearly, no graph containing K_4 is totally decomposable, but there are K_4-free graphs which are indecomposable. For example, consider the nine vertex graph with vertices $v_1, v_2, v_3, w_1, w_2, w_3, x_1, x_2, x_3$, and vertices s_i, t_j adjacent if and only if s is distinct from t. This graph corresponds to 'cloning' vertices of a triangle v, w, x by creating three nonadjacent twins for each vertex of the triangle. Assume without loss of generality that v_1 and v_2 are in W_1. Then W_2 can contain only v_3 and one other vertex; we assume that W_2 contains v_3 and w_1, which creates a $K_{2,2}$ x_1, x_2, v_3, w_1 crossing the partition.

3.2. Forbidding $2K_2$

This decomposition is of particular interest, having been studied under several different names. Hsu [8] called this the generalized join decomposition (this generalizes Cunningham's split decomposition, also known as join decomposition, which corresponds to symmetrically forbidding both $2K_2$ and P_4), and asked whether this could be found in polynomial time. The decomposition was also studied by Maire [10]; both authors were interested in this decomposition because it preserves perfection. We will show that finding a $2K_2$-free decomposition is NP-complete.

Theorem 3. *It is NP-complete to decide whether a graph has a $2K_2$-free decomposition.*

Proof. The problem of determining whether a graph has a $2K_2$-free decomposition is clearly in NP. We reduce the problem of finding a symmetric $K_{1,2}$-free decomposition to the problem of finding a $2K_2$-free decomposition.

Let G be the input graph to the symmetric $K_{1,2}$-free decomposition problem. Assuming that G is not a star (in which case we can answer that G does not have a decomposition) we create a graph G' as input to the $2K_2$-free decomposition problem as follows.

We first show that we can assume G has no vertex of degree 1. Let L be the set of vertices of G which have at least one neighbor of degree 1. Two cases can occur if $L \neq \emptyset$:

- If $|L| \geqslant 2$, or $L = \{z\}$ and z has exactly one neighbor of degree two or more, then we set $G' = G$. G has a symmetric $K_{1,2}$-free decomposition, and G' has a $2K_2$-free decomposition.
- If $L = \{z\}$ and z has at least two neighbors of degree two or more, then G has symmetric $K_{1,2}$-free decomposition if and only if the graph obtained from G by removing all its vertices of degree one has such a decomposition. Remove these vertices from G, and rename the new graph, which has no vertices of degree one, as G.

We now can assume that G has no vertex of degree one. Notice that the case where $L = \{z\}$ and z has no neighbor of degree two or more cannot occur, since G is not a star. The construction of G' is described below. Let $d(v)$ denote the degree of vertex v in G.

For each vertex v of G, create $2 * d(v)$ vertices $v_1, v_2, \ldots, v_{2d(v)}$. Connect these vertices to form a chordless cycle $v_1, \ldots, v_{2d(v)}$. Choose any order $u_1, u_2, \ldots, u_{d(v)}$ for the neighbors of v.

For each edge (w,x) of G, let i be the position of x among the neighbors of w, and let j be the position of w in the order of neighbors of x. Add edges between w_i and x_j, and between $w_{i+d(w)}$ and $x_{j+d(x)}$. We will say that these two edges of G' are *copies* of the edge (w,x) of G. Furthermore, add *crossedges* from w_i to $x_{j+d(x)}$, and from $w_{i+d(w)}$ to x_j in G'.

Finally, let (u,v), (w,x) be a pair of edges in G. If the two edges do not share any endpoints in G, take the four endpoints of the two copy edges corresponding to (u,v) in G', and connect each to all four endpoints of the two copy edges which represent (w,x) in G'. These edges will be called *filling edges*.

We now show that G has a symmetric $K_{1,2}$-free decomposition if and only if G' has a $2K_2$-free decomposition.

$\Rightarrow$: First, suppose that G has a symmetric $K_{1,2}$-free decomposition (V_1,V_2). The decomposition of G' into (V_1',V_2') is defined as follows:

$$V_1' = \{u \mid u \text{ is a copy of some vertex } x \in V_1\}$$

$$V_2' = \{v \mid v \text{ is a copy of some vertex } y \in V_2\}.$$

We will show that this decomposition is $2K_2$-free.

By contradiction, consider a $2K_2$ given by the edges (u_i,v_j), (w_k,x_l) which goes between V_1' and V_2' in G' (that is, $u_i,w_k \in V_1, v_j,x_l \in V_2$). We consider three possibilities, depending on the number of edges in the $2K_2$ which are copies of edges in G (two, one or zero edges).

Case 1. Both (u_i,v_j) and (w_k,x_l) are copies of edges of G. First, suppose that u_i and w_k are copies of the same vertex. Since every vertex in V_1 has at most one neighbor in V_2, v_j and x_l are also copies of the same vertex, i.e. these are two copies of the same edge. This cannot occur, since our construction adds crossedges between copies of the same edge. Thus, all four endpoints correspond to distinct vertices. But in this case, the construction adds all filling edges between the two edges. We have a contradiction.

Case 2. The edge (u_i,v_j) is a copy of an edge of G, but (w_k,x_l) is not. So (w_k,x_l) is a filling edge or a crossedge.

If (w_k,x_l) is a filling edge, there exist edges (b,x), (w,c) of G with all four endpoints distinct. Now, at least one edge (between u_i and x_l) is missing in G' between the endpoints corresponding to (b,x) and (u,v). So, these edges must share a common point (else all filling edges would have been added): either $b=u$ or $x=v$. Since the decomposition of G leaves only a matching between V_1 and V_2, in either case we must have both $b=u$ and $x=v$. Similarly, we would require $w=u$ and $c=v$; since there are only two copies of any edge, this case cannot occur.

If (w_k,x_l) is a crossedge, then it was added between two copies of the same edge (w,x). Since edges between V_1 and V_2 form a matching, neither u nor v can be equal to w or x; thus, we would add all edges between the corresponding endpoints, violating the assumption that these form a $2K_2$ in G'.

Case 3. Neither (u_i,v_j) nor (w_k,x_l) are copies of edges of G.

First, suppose that both edges are filling edges. Say that (u_i,v_j) was added because of edges (u,d) and (e,v) in G, and that (w_k,x_l) was added because of edges (w,f) and (g,x) in G. Since we did not add all edges between (u,d) and (g,x), some endpoint must be shared; we may assume that $u=g$. Since (u,d) and (u,x) both go from V_1 to

V_2 in G, we must have $d=x$. Since (u,d) and (g,x) are copies of the same edge, by construction we should have added a crossedge from u_i to x_l.

Our only remaining case is that at least one of the edges of the $2K_2$ (let us say (u_i,v_j)) is a crossedge, i.e. it was added between copies of a single edge (u,v). Since neither u nor v can have another neighbor on the other side of the partition, any edges of G which caused (w_k,x_l) to be added cannot have u or v as an endpoint; thus, crossedges should have been added, contradicting the assumption that these edges form a $2K_2$.

$\Leftarrow$: Suppose that G' has a partition (V_1',V_2') such that no $2K_2$ exists between V_1' and V_2'. We will show that G has a symmetric $K_{1,2}$-free decomposition.

We divide this proof into two cases. In the first case, we assume that for every vertex v of G, all copies of v are on the same side of the partition of G'. In the second case, we assume that there is a vertex of G such that some copy of v occurs on each side of the partition of G'.

Case 1. Suppose that all copies of every vertex lie on a single side of the partition of G'. Then let the partition of G be

$$V_1=\{x \mid \text{all copies of } x \text{ are in } V_1'\},$$

$$V_2=\{y \mid \text{all copies of } y \text{ are in } V_2'\}.$$

We will show that (V_1,V_2) is a symmetric $K_{1,2}$-free decomposition of G.

On the contrary, we may assume that there is a vertex x in V_1 which has two neighbors y,z in V_2. In G', the copies (x_i,y_k) of (x,y) and (x_j,z_l) of (x,z) go from V_1' to V_2', and since (x,y), (x,z) share an endpoint of G, the construction will not add any edges between the corresponding endpoints in G'; thus we contradict the assumption that V_1',V_2' is a $2K_2$-free decomposition.

Case 2. Assume that there is some vertex v of G which has copies in both V_1' and V_2' in the decomposition of G'. Let our partition have the minimum number of vertices which have copies in both V_1' and V_2'.

The copies of v form an induced cycle in G'. We claim that the only way to split an induced cycle $c_1,\ldots,c_k$ into two pieces of a $2K_2$-free decomposition is to put one vertex on one side, and all other vertices on the other side. Otherwise, call *segment* of the cycle every maximal set of successive vertices along the cycle, such that all the vertices are on the same side. Now, the cycle cannot have two segments $c_i,\ldots,c_j$, $c_k,\ldots,c_l$ in V_1 since the edges (c_j,c_{j+1}) and (c_l,c_{l+1}) violate the $2K_2$-free condition. If it has only one segment $c_i,\ldots,c_j$ in V_1, then $j=i$ or else (c_{i-1},c_i) and (c_j,c_{j+1}) give a $2K_2$.

Thus, we may assume that v_i is in V_1', and all other copies of v are in V_2'. Placing v_i in V_2', and changing the position of no other vertex, gives a partition of G' which cannot be a $2K_2$-free decomposition, otherwise the decomposition (V_1',V_2') would not have the minimum number of vertices with copies on both sides of the partition. We have two possibilities.

- Either $V_1' - \{v_i\}$ contains only one vertex w_j; then, since the decomposition (V_1', V_2') is $2K_2$-free, we must have either $N_{V_2'}(v_i) \subseteq N_{V_2'}(w_j)$ or $N_{V_2'}(w_j) \subseteq N_{V_2'}(v_i)$. Assume that the first holds and call t_p the neighbor of v_i corresponding to an edge vt of G. Now, $w_j \neq t_p$ since (t_p, v_{i-1}) is not an edge. So (w_j, t_p) is an edge of G'. This can be neither a copy of an edge of G (t_p is in only one such copy), nor a crossing edge (w_j would be a copy of v_i), so (w_j, t_p) is a filling edge corresponding to edges (v, t), (w, y) of G. But then by construction, v_i is adjacent to $w_{j+d(w)}$ (the endpoint of the other copy of (w, y) in G', while w_j is nonadjacent to $w_{j+d(w)}$. Then $N_{V_2'}(v_i) \subseteq N_{V_2'}(w_j)$ is contradicted.
- or the partition $(V_1' - \{v_i\}, V_2' \cup \{v_i\})$ contains a $2K_2$, since this reduces the number of vertices which have copies in both sides of the partition. Thus, there must be some w_j in V_1' such that (v_i, w_j) is an edge of G'. By our construction, $(v_{i+d(v)}, w_j)$ is also an edge of G'. The edges (v_{i-1}, v_i) and $(v_{i+d(v)}, w_j)$ from a $2K_2$ in G', violating the assumption that we have a $2K_2$-free decomposition of G'. $\square$

Considering graphs which are totally decomposable with respect to $2K_2$-free decomposition yields a subclass of weakly triangulated graphs (i.e graphs with no chordless cycle of length at least five and with no complement of such cycle) and a superclass of domination graphs (i.e graphs in which every induced subgraph has two vertices with comparable neighborhoods). We did not succeed in identifying this class, but we discuss some related aspects in a paper devoted to domination graphs [13].

4. Conclusions and open problems

We have defined a new form of graph decomposition, and studied some important special cases. We have shown that it is NP-complete to determine whether a graph has a generalized join.

One motivation for studying the matching and generalized join decompositions comes from their perfection preserving properties. The generalized join preserves perfection (see [7,10]), i.e. if V_1 and V_2 induce perfect graphs, then G is a perfect graph too. For the 'matching' decomposition, the result is a little bit weaker, in the sense that we can only claim that no minimal imperfect graph different from a C_{2k+1} ($k \geqslant 2$) has a symmetric $K_{1,2}$-free decomposition. For this, it is sufficient to notice that the intersection graph of the maximum cliques of G is disconnected (no maximum clique can have vertices in both V_1 and V_2), and this is not possible by Preissmann's result [12].

Identifying the totally decomposable graphs with respect to the matching decomposition and to the generalized join would therefore be interesting, from the point of view of perfect graph theory.

On the other hand, it would be interesting to characterize completely when a decomposition with a forbidden subgraph is polynomial or NP-complete. A particular open case is the P_4-decomposition.

Furthermore, it seems likely that the time complexity for $K_{1,i}$-free decomposition, i.e. allowing each vertex in V_1 to have at most $i-1$ neighbors in V_2, can be improved considerably.

Applications of these decompositions would be interesting to find, especially for those which can be found in polynomial time.

5. Uncited references

[6,14]

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